Reed Solomon Erasure Coding for the Optimization of Network Reliability and Energy Efficiency in Wireless Sensor Networks

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Abstract—
This Network error correction (NEC) helps to correct errors of a communication network by using the concept of network coding [1] and provides an alternative to the traditional link-by-link error correction. NEC is widely used in sensor networks and in video network broadcasting, where the network suffers from delay and energy constraints. This paper aims to design an Automatic Repeat Request (ARQ)-based RS-erasure code to correct the errors/erasures encountered in the data packets due to the random errors and unreliable network links. The proposed protocol tries to combine the RS-Erasure code and Selective Repeat (SR-based) retransmission scheme to achieve the higher network reliability and energy efficiency in wireless sensor networks. In this scheme the code rate of a given message block is calculated dynamically depending upon the error rate of the link to minimize the total transmission efforts. The performance of the proposed technique is compared with some existing techniques like SR-ARQ, SRTP [11] and CRBT [24] on the basis of Total transmissions required for end to end reliable data delivery, energy consumption by the network nodes, transmission delay and the network-throughput.

Keywords— Include Network Coding, Network Error Correction, ARQ efficiency, Reliability, Erasure code, Reed Solomon Code

I. INTRODUCTION

Data transmission within a network can be affected by the presence of errors or erasure of symbols which can be originated from various sources like channel noise, bit flipping or introduction of inappropriate messages by the malicious network nodes. Unavoidable network congestion and traffic jam can also lead to the loss of packets. The conventional error correcting codes add some redundancy to the source message for data protection in time domain. There are some another mechanisms which add redundancy to the transmission in special domain for error correction.

This Network error correction (NEC) was designed to recover the erroneous symbols and lost packets of the network by expanding the potential of classical error-correcting codes [2][3]. NEC includes all the basic concepts and features of classical coding theory such as weight measures, coding bounds and the coding distance which considers a network as a coding operator. The normal form of a network coding can be treated as a special case of Network error correction NEC without considering its error control properties. While the wide area of its applications can be found in the field of sensor networks and video broadcast networks where the network nodes are delay sensitive and suffers from energy constrains.

A network in which all the characteristics (topology and network code) are known in advance to both sender and receiver, prior to the data transmission is known as coherent networks while in non-coherent networks, both sender and receiver are unaware about the network characteristics. In these models, there exist some additional linear MDS codes which help in limiting the errors to a great extent.

II. RELATED WORK

The basic concept of classical error correction has been extended by some researchers into network error correction with the study of algebraic coding. Researchers also explored the concept of reliable transmission with network coding in the case of imperfect links [4][5][6]. A structure of coherent network error correction [7] is presented with the assumption of knowing the network topology and network code by the sinks nodes. Yet another model of noncoherent network error correction [8]-[11] has been proposed, which do not rely on this assumption. For the networks operating in multicast scenarios, several effective polynomial-time algorithms [12] for the construction of network code are proposed. Whereas for the non-multicast [13] scenarios, detection of
such efficient network codes is a major issue. Lehman [14] focused on the complexity of various problems of network coding.

III. GENERAL ERROR CORRECTIONS

The packet networks [15][16] specially designed for the non coherent network error correction, explores the algebraic definition of the minimum distance of a code. The authors of [17] show the relation between network coding and maximum distance separation (MDS) codes in the classical algebraic coding theory. Codes to deal the lost packets and erasures, has been explored with the help of inter Packet level redundancy schemes [18][19]. In these schemes the source node encodes m number of packets into m+k encoded packets [20] form which the receiver can successfully recover the original data. Vuran [21] considered the efficient data collection in case of single hop UWAN (underwater acoustic network) from multiple underwater nodes. In this scheme the nodes overhear the transmissions from other nodes, which is helpful in delivering the missing packets to the destination in network coding environment.

Some time due to congestion or network jamming, the entire link breaks and the packets are lost/erased. Due to these insufficient linearly independent packets, the entire generation cannot be decoded completely. If the local encoding function of a node depends on the output of that channel, the code assumes that a default symbol in the finite field $F$ is received on that channel at the time of erasure. So associated with each channel is a default symbol. If a network in which only erasures can occur in channels and the sink nodes have knowledge of all erasure channels but has no idea of default symbols, then it is simply assumed that a random symbol will be transmitted in the channel at the time of erasure occurrence. If the sink node $t$ has information about the message set $C$ and the transfer matrices $F_{st}$ and $F_t$, then for the erasure correction, the below mentioned quantities are considered.

A. Error detection and Correction Capability of a Code

The ability of a code for the identification and correction of errors is heavily depends upon the type of decoder. In a joint framework, where both error correction and detection is made, a codeword of the code is chosen as a message and is transmitted via a channel. Along with each codeword, a subset of $\Phi$ is attached which is known as the encoding region for the codeword.

A decoder will produce a correct message when the received vector lies in the decoding region of a particular codeword, and this error vector is known as correctable. On the other side, when the received vector does not lie in the decoding region, the error vector is known as detectable only and in this situation either the decoder outputs a correct message or a warning about the occurrence of an error (error detection).

B. Network Erasure Correction

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- The Hamming weight of an erasure pattern $\rho$, denoted by $|\rho|$.
- The Network Hamming weight of an error pattern $\rho$, It is defined as
  $$W_t^{esr}(\rho) = \max_{\rho^{*}} W_{\rho^{*}}$$  
  (i)

  Where $\rho^{*}$ refers to the set of erasure vectors that matches $\rho$.

  As $W_t^{esr}(z) \leq W_\rho(z) \leq |\rho|$ for any $z \in \rho^{*}$ then
  $$W_t^{esr}(\rho) = \max_{\rho^{*}} W_{\rho^{*}}(\rho)$$  
  (ii)

The following features of a linear network code are equivalent for a non-negative integer $d$.

- The code is capable of correcting any erasure pattern $\rho$ with $|\rho| \leq d$ at all the sink nodes.
- A code is capable to correct any erasure pattern $\rho$ with $W_t^{esr}(\rho) \leq d$ at all the sink nodes.
- A code must have $d + 1 \leq d_{min}$.

IV. REED-SOLOMON (RS) PACKET LEVEL ERASURE CODE

Error correction capability of a code is based upon the metrics or the distance function chosen. In algebraic theory, although the widely used metric is hamming but it is not suitable when the symbol of higher dimensions Galois field is used. To solve this problem, Maximum Rank Distance (MRD) codes are preferred. Reed-Solomon is the best known cyclic error correcting code for its excellent error correction capability and resilience.
against packet losses. This code is optimized for the correction of burst errors and provides a genuine trade-off between code efficiency and its computational complexity. Although the code rate of a Reed Solomon code is high but its decoding complexity is also slightly high.

Erasure codes can be classified into optimal erasure and near-optimal erasure codes [22]. But the RS code belongs to the optimal erasure codes [23], in which the receiver can decode the original data packets from any subset of \( m \) packets transmitted from the sender side. In Reed Solomon coding, the required redundancy packets for the successful decoding of the packets at the receiver is directly related to the Packet error rate from sender. Where \( k \) makes the decision of packet retransmission for the receiver recovers the \( k \) number of source elements from any set of \( k \) received (ly more into \( k \) distinct segments i.e. encoded data packets are transmitted to the next error free (he optimal erasure codes (e.g. RS code) [24] with some other schemes like Selective repeat (SR-ARQ), Segmented reliable Transport protocol [11] and Coding Based Reliable Transmission (CRBT) [24] on the account of following parameters:

A. Description of Proposed Scheme

Restricting Error propagation (REP-NC) scheme is based upon the combination of RS erasure codes and Selective Repeat (SR) retransmission scheme. In REP-NC, the source node generates \( m+n \) encoded packets form the \( m \) number of source packets so that the original data can be reconstructed from any subset of \( m \) encoded packets. From the source node, these encoded data packets are transmitted to the next error free node of the network. During the transmission if any symbol error has encountered, then the proposed protocol restricts the Propagation of Errors by applying REP-NC algorithm.

At each node the exact number of errors and its positions are recorded. If numbers of errors encountered are lesser the maximum error correction capacity (MECC) of proposed Error Correction Coding System and at that node network coding is not applied then faulty packet is moved forward without any error correction, but if the node requires to combine the packet with some other packets of the network, then before combination of these distinguish packets, encountered error will be corrected first so that it should not degrade the network performance. When encountered errors reach the MECC, error correction takes place. Negative acknowledgement (NAK) signal from receiver end intimates the corresponding sender about the missing number of data packets. From this NAK, the sender takes the decision of packet retransmission for the successful packet decoding at receiver side.

B. RS Encoding Algorithm

The encoding of RS \( (n,k) \) code over the \( GF(q) \) is as follows:

Let \( \alpha_1, \alpha_2, ..., \alpha_n \) be the \( n \) number of discrete non-zero elements of the \( GF(q) \), where \( q > n \).

Step 1: Divide the message \( \vec{M} \) into \( k \) distinct segments i.e. \( M_1, M_2, ..., M_k \).

Step 2: Devise the polynomial \( M(x) \) where \( M(x) = \sum_{i=0}^{k-1} M_i x^i \).

Step 3: Calculate the value of \( M(x) \) with \( x = \alpha_i \) for \( i = 1,2,...,n \).

Step 4: Define the encoded vector \( \vec{E} = \{ M(\alpha_1), M(\alpha_2), ..., M(\alpha_n) \} \).

Step 5: Calculate the \( M_{dist} \) of two distinct code words i.e. \( \vec{e}_1 \) and \( \vec{e}_2 \). Where \( M_{dist} \) represents the minimum distance for outputting the minimum number of positions the vector \( \vec{e}_1 \) and \( \vec{e}_2 \) differ.

If related to \( \vec{e}_1 \) and \( \vec{e}_2 \), \( M_1(x) \) and \( M_2(x) \) are the two polynomials of degree \( (k-1) \), then the difference \( M_1(x) - M_2(x) \) will produce a polynomial having at most \( (k-1) \) roots. It states that the codewords \( \vec{e}_1 \) and \( \vec{e}_2 \) are similar at the most \( (k-1) \) locations and differs at \( (n - (k - 1)) \) i.e. \( (n - k + 1) \) locations, which is denoted as the minimum distance \( (d_{min}) \) of RS. If the \( d_{min} \) value of a RS-code is large, it can correct more errors.

C. RS Decoding Algorithm

RS decoding is based upon the basic property of the generator matrix, which states that any \( k \times k \)-submatrix is invertible. In this process, the receiver recovers the \( k \) number of source elements from any set of \( k \) received elements.

Step 1: Extract the sub-matrix (of dimension \( k \times k \)) of the generator matrix \( G \).

Step 2: Invert this matrix.

Step 3: To recover the source vector, multiply the newly received vector by the obtained matrix.

V. REED-SOLOMON (RS) PACKET LEVEL ERASURE CODE

This section evaluated the performance of the proposed REP-NC technique and the results are compared with some other existing schemes like Selective repeat (SR-ARQ). Segmented reliable Transport protocol [11] and Coding Based Reliable Transmission (CRBT) [24] on the account of following parameters:
- Total transmissions required for end to end reliable data delivery.
- Energy consumption by the network nodes
- End to end Transmission delay
- Network-Throughput

The proposed protocol is implemented in MATLAB with the assumption of single source node containing 220 source data packets, each having size of 60 bytes. Further, in each block, four packets are grouped together. In this work total 55 original data blocks are taken and the successful packet decoding probability is 95% (i.e., δ = 0.05). The control signal for each ACK/NACK packet is of 6 bytes. The simulation was carried out for 1000 seconds and the result is the average of 30 rounds.

In the SR-ARQ, only the selected lost packets are retransmitted from the sender side window (of size 5) after waiting for the ACK of each packet. In this scheme a large window is maintained at the sender side. The packets form this window is transmitted to the destination end till it receives the positive acknowledgement. On the reception of NAK, packets are retransmitted.

In Figure 1, the graph shows the relation of total packets delivered with reference to the hop count by considering the average erasure rate of 0.4 for each source-destination pair.

![Figure 1. Variation in Hop Count to the total number of packets sent](image)

It has been found that with the increase of hop count, the number of packets to be sent is also increases. However when the performance of the proposed technique is compared with other schemes, the number of packets transferred is less with the proposed one. This is due to that fact that the reliable transmission in SR-ARQ is purely dependent upon the retransmission of the lost packets. In SRTP, a stream of encoded packets is sent from the sender side till the reception of ACK form receiver. Moreover at the time of retransmission, the sender does not know the count of packets required by the receiver for successful packet decoding. The CBRT scheme is based on RLNC and requires more retransmissions for successful recovery of the original message (m packets) from the set of all the (m+k) coded packets at the destination side. However the proposed hybrid ARQ scheme computes the code rate dynamically on the basis of packet error rate of each link. So, requires less retransmission efforts.

![Figure 2. Packet Erasure Probability Vs Total number of packets sent](image)

In Figure 3, the effect of packet erasure probability on the total packets transmitted is shown. In all the other schemes, with the increase of erasure rate more number of packets is required for the successful data delivery but in the proposed scheme this count is less due to the adaptive computation of coding ratio.

Next Figure 3 specifies the variation in packet size to the total number of packets required. The packet length is variable and can attain any size ranging from 30 – 80 bytes. In the proposed hybrid scheme the source
node contains 220 packets of 60 bytes each. When the packets are of larger size, they are more susceptible to errors and leads to packet losses, so more retransmissions are demanded. But with the small sized packets, the transmission delay increases and the network incur computation overhead. As the proposed scheme has moderate packet size so fewer packets are transferred.

As the distance between the hops increases, it will increase the total transmission time and energy consumption of nodes and ultimately lowers the channel bandwidth. So more power is consumed for the end to end transmission.

Fig. 3. Variation in Packet Size to the total number of packets sent

In Figures 4, a plot between average delay for transmitting a message block against the number of hops in the network and erasure probability is depicted. When the count of packets to be retransmitted is more, the total propagation and modem conversion delays are also more.

Fig. 4. Average end to end delay per message Block with the change of Erasure Probability

Figure 5 shows the relation of throughput and erasure rate for the various schemes. Throughput can be judged from the data successfully received at the destination end in one second and is a function of block size of the message. As the erasure rate increase, the throughput decreases. As compared to all other schemes the throughput gain of proposed technique is higher due to lower packet drop ratio and fewer redundant transmissions.
VI. CONCLUSIONS

In this paper the proposed scheme is based on the combination of Reed Solomon erasure coding and the selective retransmission scheme for ensuring the end to end data delivery in Wireless Sensor Networks operating in the network coding environment. The simulation for the proposed scheme has been carried out in MATLAB and the result shows that, the performance of Hybrid RS-Erasure ARQ technique is better than SR-ARQ, SRTP and CRBT techniques on the basis of total transmissions required for end to end reliable data delivery, energy consumption by the network nodes, transmission delay and the network-throughput.

REFERENCES

