Dynamical Behavior of Discrete Prey-Predator system with Scavenger

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Abstract: This paper deals with the prey – predator system with scavenger. The model is developed with difference equations which is suitable when the population has non overlapping generations. Stability analysis is carried out for the equilibrium points. Time series plots and bifurcation diagrams are provided.

Keywords: Discrete prey predator model, scavenger, stability, time plots, bifurcation.

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I. INTRODUCTION

Ecology is the study of inter-relationships between organisms and environment. It is natural that two or more species living in a common habitat interact in different ways. The application of mathematics to ecology dates back to the book “An Essay on the Principle of Population by Malthus” [6]. Verhulst (1838) introduced a population model with limited resources by an equation known as the logistic equation. The first mathematical model describing interacting populations were formulated by Lotka and Volterra [4] independently. The Lotka – Volterra system allows drawing important conclusions regarding qualitative behavior of the system. But the system is structurally unstable. This paper deals with the food web consisting of three interacting species with scavenger. Scavenger is an organism that mostly consumes decaying meat. Scavengers are a part of the food web and it plays an important role in the food web. They keep an ecosystem free of the bodies of dead animals or carrion. Scavengers break down this organic material and recycle it into the ecosystem as nutrients. Some birds (Vultures), many insects (Blowflies), many mammals (Hyenas), some sea creature (crabs, lobsters) and many animals (Lions, Leopards and Wolves) are scavengers. In 1877, naturalist Sebastian Tenney enjoined his colleagues to devote more study to scavenger. But scavengers have long been neglected in population ecology, relative to herbivores and predators [3]. In recent years, the field of systems ecology has placed great emphasis on the study of detritus and detritrivores. Several mathematical models have been developed to model detritus and detritrivore abundance in ecosystems.

II. MODEL DESCRIPTION AND EQUILIBRIUM POINTS

We consider the presence of scavenger species in a logistic Lotka - Volterra model as described by the following system.

\[ \begin{align*}
  x(n+1) &= r x(n)[1 - x(n)] - x(n) y(n) \\
  y(n+1) &= (1-a) y(n) + x(n) y(n) \\
  z(n+1) &= (1-b) z(n) + c x(n) y(n) z(n) + d x(n) z(n) + e y(n) z(n)
\end{align*} \] (1)

Where \( r, a, b, c, d, e > 0 \). The system (1) has the three equilibria \( E_0 = (0,0,0); \ E_1 = (1-\frac{1}{r}, 0,0); \) and \( E_2 = (a, r(1-a)-1,0) \)

III. STABILITY ANALYSIS AND NUMERICAL STUDY

The recent decades have witnessed development of theory of non linear dynamical systems having applications in various fields including population biology. Due to the complexity in analyzing non linear equations, a comprehensive theory of dynamics of non linear systems still awaits development. An important technique for analyzing nonlinear systems qualitatively is the analysis of the behavior of the solutions near equilibrium points using linearization. One of the most urgent problems demanding the attention of researchers is the stability of ecosystems. The local stability analysis of the model can be carried out by computing the Jacobian matrix corresponding to each equilibrium point. The Jacobian Matrix \( J \) for the system (1) is

\[ J(x, y, z) = \begin{pmatrix} r - 2rx - y & -x & 0 \\ y & 1 - a + x & 0 \\ cyz + dz & cxz + ez & 1 - b + cxy + dx + ey \end{pmatrix} \] (2)

Analytical studies always remain incomplete without numerical verification of the result. In this section, we present the time plots of prey, predator and scavenger, phase portraits and bifurcation diagrams for the system (1) obtained in previous sections. The numerical experiments are designed to show the rich complex dynamical behavior of the system with different set of parameters. We need the following Lemma to discuss the stability of the equilibrium points of (1).
Lemma 1. Let
\[ p(\lambda) = \lambda^3 + A_1 \lambda^2 + A_2 \lambda + A_3 = 0 \]  \hspace{1cm} (3)
be the characteristic equation for a matrix defined by (2). Then the following statements are true:

1. If every root of equation (3) has absolute value less than one, then the equilibrium point of the system (1) is locally asymptotically stable and equilibrium point is called a sink.
2. If at least one of the roots of equation (3) has absolute value greater than one, then the equilibrium point of the system (1) is unstable and equilibrium point is called a saddle.
3. If every root of equation (3) has absolute value greater than one, then the equilibrium point of the system (1) is a source.
4. The equilibrium point of system (1) is called hyperbolic if no root of equation (3) has absolute value equal to one. If there exists a root of equation (3) with absolute value equal to one, then the equilibrium point is called non-hyperbolic.

Using the lemma, we have the following results for the system (1). The propositions are followed by examples of Numerical simulation of stability of equilibrium points.

Proposition 1. The equilibrium point \( E_0 \) is a

(a) sink if \( r < 1, 0 < a < 2 \) and \( 0 < b < 2 \);
(b) source if \( r > 1, a > 2 \) and \( b > 2 \);
(c) saddle if \( r > 1, 0 < a < 2 \) and \( 0 < b < 2 \);
(d) non hyperbolic if \( r = 1 \) or \( a = 2 \) or \( b = 2 \).

Proof. Jacobian matrix for \( E_0 \) is given by
\[
J (E_0) = \begin{pmatrix} r & 0 & 0 \\ 0 & 1-a & 0 \\ 0 & 0 & 1-b \end{pmatrix}
\]
The eigenvalues of the matrix \( J (E_0) \) are \( \lambda_1 = r, \lambda_2 = 1-a \) and \( \lambda_3 = 1-b \). By Lemma, it is easy to see that, \( E_0 \) is a sink if \( r < 1, 0 < a < 2 \) and \( 0 < b < 2 \); \( E_0 \) is a source if \( r > 1, a > 2 \) and \( b > 2 \); \( E_0 \) is a saddle if \( r > 1, 0 < a < 2 \) and \( 0 < b < 2 \); and also \( E_0 \) is a non hyperbolic if \( r = 1 \) or \( a = 2 \) or \( b = 2 \).

Example 1. We shall consider the values \( r = 0.99, a = 0.099, b = 0.95, c = 0.095, d = 0.5, e = 0.8 \). From the time plot and phase portrait the trajectory towards the origin, see figure – 1(a, b)
Proposition 2. The equilibrium point $E_1$ is a

(a) sink if $1 < r < 3, r < \frac{1}{1-a}$ and $r < \frac{d}{d-b}$;

(b) source if $r > 3, r > \frac{1}{1-a}$ and $r > \frac{d}{d-b}$;

(c) saddle if $r > 3, r < \frac{1}{1-a}$ and $r < \frac{d}{d-b}$.

Proof. Jacobian matrix for $E_1$ is given by

$$
J(E_1) = \begin{pmatrix}
2-r & \frac{1}{r} & 0 \\
0 & 2-a & \frac{1}{r} \\
0 & 0 & 1-b+d\left(\frac{1}{r}\right)
\end{pmatrix}
$$

The eigenvalues of the matrix $J(E_1)$ are $\lambda_1 = 2-r, \lambda_2 = 2-a - \frac{1}{r}$ and $\lambda_3 = 1-b+d\left(\frac{1}{r}\right)$. By using Lemma, it is easy to see that, $E_1$ is a sink if $1 < r < 3, r < \frac{1}{1-a}$ and $r < \frac{d}{d-b}$; $E_1$ is a source if $r > 3, r > \frac{1}{1-a}$ and $r > \frac{d}{d-b}$; and finally $E_1$ is a saddle if $r > 3, r < \frac{1}{1-a}$ and $r < \frac{d}{d-b}$.

Example 2. We shall consider the values $r = 1.65, a = 0.55, b = 0.45, c = 0.85, d = 0.95, e = 0.15$. The equilibrium point $E_1 = (0.39, 0, 0)$ and the eigen values are $\lambda_1 = 0.35, \lambda_2 = 0.8439$ and $\lambda_3 = 0.9242$. Also $|\lambda_{2,3}| < 1$ which is Stable, see figure – 2(a, b). We shall consider the values $r = 3.25, a = 0.7, b = 0.7, c = 0.95, d = 0.8, e = 0.8$. The eigen values are $\lambda_1 = -1.25, \lambda_2 = 0.99$ and $\lambda_3 = 0.8538$. Here $|\lambda_1| > 1$ and $|\lambda_{2,3}| < 1$, which is unstable. See figure – 3(a, b).
Proposition 3. The equilibrium point $E_2$ is a

(a) sink if $\frac{1}{1-a} < r < \frac{b+e+a(c-d)}{(1-a)(ac+e)}$;

(b) source if $\frac{b+e+a(c-d)}{(1-a)(ac+e)} < r < \frac{1}{1-a}$;

(c) saddle if $r > \frac{b+e+a(c-d)}{(1-a)(ac+e)}$ and $r > \frac{1}{1-a}$.

Proof. From (2), Jacobian matrix for $E_2$ is given by

$$J(E_2) = \begin{pmatrix} 1-ar & -a & 0 \\ r(1-a)-1 & 1 & 0 \\ 0 & 0 & 1-b+a[rc(1-a)-re+(d-c)+e(r-1)] \end{pmatrix}$$

Hence the eigenvalues of the matrix $J(E_2)$ are $\lambda_1 = 1-b-e-a(c-d)+r(e+ac)(1-a)$ and $\lambda_{2,3} = 1 - \frac{ar}{2} \pm \frac{1}{2} \sqrt{ar(ar-4)+4a(ar+1)}$. By using Lemma, we conclude that, $E_2$ is a sink if $1 < r < \frac{b+e+a(c-d)}{(1-a)(ac+e)}$; $E_2$ is a source if $\frac{b+e+a(c-d)}{(1-a)(ac+e)} < r < \frac{1}{1-a}$; and finally $E_2$ is a saddle if $r > \frac{b+e+a(c-d)}{(1-a)(ac+e)}$ and $r > \frac{1}{1-a}$.

Example 3. We shall consider the values $r = 3.99, a = 0.55, b = 1.25, c = 0.9, d = 0.15, e = 0.8$. The equilibrium point $E_2 = (0.55, 0.79, 0)$ and the eigenvalues are $\lambda_1 = 0.8625, \lambda_2 = 0.7781$ and $\lambda_3 = -0.9727$. Also $|\lambda_{2,3}| < 1$ which is Stable,
see figure – 4(a, b). We shall consider the values \( r = 4.09, a = 0.55, b = 0.85, c = 0.01, d = 0.05, e = 1.05 \). The eigenvalues are \( \lambda_4 = 1.0647, \lambda_2 = 0.7713 \) and \( \lambda_3 = -1.0207 \). Here \( |\lambda_2| < 1 \) and \( |\lambda_3| > 1 \), which is unstable, See figure – 5(a, b).

Figure 4(a): Time series plot for Stability at equilibrium point \( E_2 \)

Figure 4(b): Phase Portrait at equilibrium point \( E_2 \)

Figure 5(a): Time series of Unstable equilibrium point \( E_2 \)
A bifurcation is a qualitative change in the behavior of solutions as one or more parameters are varied. The parametric values at which these changes occur are called bifurcation points. If the qualitative change occurs in a neighborhood of a fixed point or periodic solution, it is called a local bifurcation. Any other qualitative change that occurs is considered as a global bifurcation.

Example 4. The parameters are assigned the values $a = 0.65$, $b = 1.25$, $c = 0.95$, $d = 0.15$, $e = 0.8$ and the bifurcation diagram is plotted for the growth parameter in the range 1.5 – 4. Both prey and predator population undergoes chaos, see Figure - 6.

In this paper, ecological model with interspecies interactions in three species food chain with prey - predator and scavenger is proposed and studied some dynamical behaviors are investigated. Dynamical behavior of three species food chain discrete model is investigated at equilibrium points. Numerical study shows the rich and interesting complicated dynamics of the model. Also the bifurcation diagram is provided.

REFERENCES


